Time Series

Project II

Testing for the Validity of PPP

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1 Purchasing Power Parity

Purchasing power parity is an economic theory which states that in the presence of international trade, the price of a single good will be the same across countries, under certain assumptions. The log specification is as follows:

$$p_t(j) = p_t^*(j) + S_t \tag{1}$$

where $p_t(j)$ is the price of good j, $p_t^*(j)$ price of the same good (j) and S_t is the nominal exchange rate. The relationship that we are interested in incorporates all traded and non-traded goods and the equation is:

$$log\hat{S} = log\hat{P}_t^D - logP_t^F \tag{2}$$

where $\hat{P}_t^D$ is domestic price level, P_t^F is foreign price level and $\hat{S}$ is nominal exchange rate. This paper will test for the validity of Purchasing Power Parity using data of United States(U.S.) and Germany following three different methods:

- Bivariate Specification
- Univariate Specification
- Cointegration Analysis

2 Data Collection

The time series monthly data from May 1972 to October 2017 used for the analysis are collected from Federal Reserve Economic Data (FRED). For price level consumer price index (index 2015 =100) is used for both countries. While the available consumer price index(CPI) of the U.S. was seasonally adjusted, other required data were not and R was used to seasonally adjust CPI and exchange rate. It should be noted that starting from 1999 German Currency (DM) was fixed at the rate of 1 €= 1.95583 DM and I used this conversion rate to get US against DM exchange rate after 1998. The prior years' data were collected from FRED as mentioned earlier.

Figure (1) and Figure (2) shows the plot of seasonally adjusted and unadjusted data of CPI and Germany(CPI_G) and nominal exchange rate respectively. R codes to seasonally adjust CPI of Germany and the corresponding plot are:

```
> library(readxl)
> library('ggplot2')
> library("foreign")
> library("forecast")
> library('tseries')
> CPIG <- read_excel("G:/mahmood/DAL/Time series/Project two/CPIG.xls")
> CPIG<-CPIG[-c(710:719),]
```

```
> CPIG$Time = as.Date(CPIG$Time)
> count_ts <- ts(CPIG[, c('CPI_G')])
> CPIG$clean_cnt <- tsclean(count_ts)
> CPIG$cnt_ma30 <-ma(CPIG$clean_cnt, order=30)
> ggplot() +
+   geom_line(data = CPIG, aes(x = Time,
+             y = clean_cnt, colour = "no adjustment")) +
+   geom_line(data = CPIG, aes(x = Time,
+             y = cnt_ma30, colour = "adjusted"))  +
+   ylab('CPI_G')
```

Rcodes to seasonally adjust exchange rate and the corresponding plot are:

```
> library(readxl)
> library('ggplot2')
> library("foreign")
> library("forecast")
> library('tseries')
> DMto1US <- read_excel("G:/mahmood/DAL/Time series/Project two/DMto1US.xlsx")
> DMto1US<-DMto1US[-c(577:579),]
> DMto1US$Time = as.Date(DMto1US$Time)
> count_ts = ts(DMto1US[, c('USagDM')])
> DMto1US$clean_cnt = count_ts
> DMto1US$cnt_ma30 = ma(DMto1US$clean_cnt, order=30)
> ggplot() +
+   geom_line(data = DMto1US, aes(x = Time,
+           y = clean_cnt, colour = "no adjustment")) +
+   geom_line(data = DMto1US, aes(x = Time,
+         y = cnt_ma30, colour = "with adjustment"))  +
+   ylab('$againstDM')
```

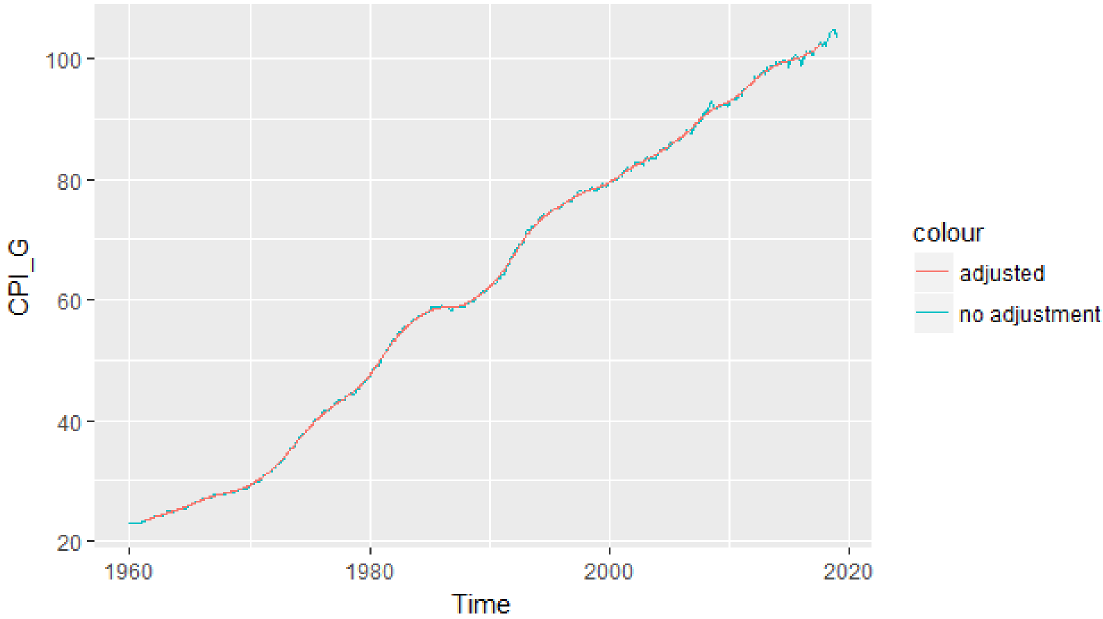

Figure 1: comparison of CPI_G

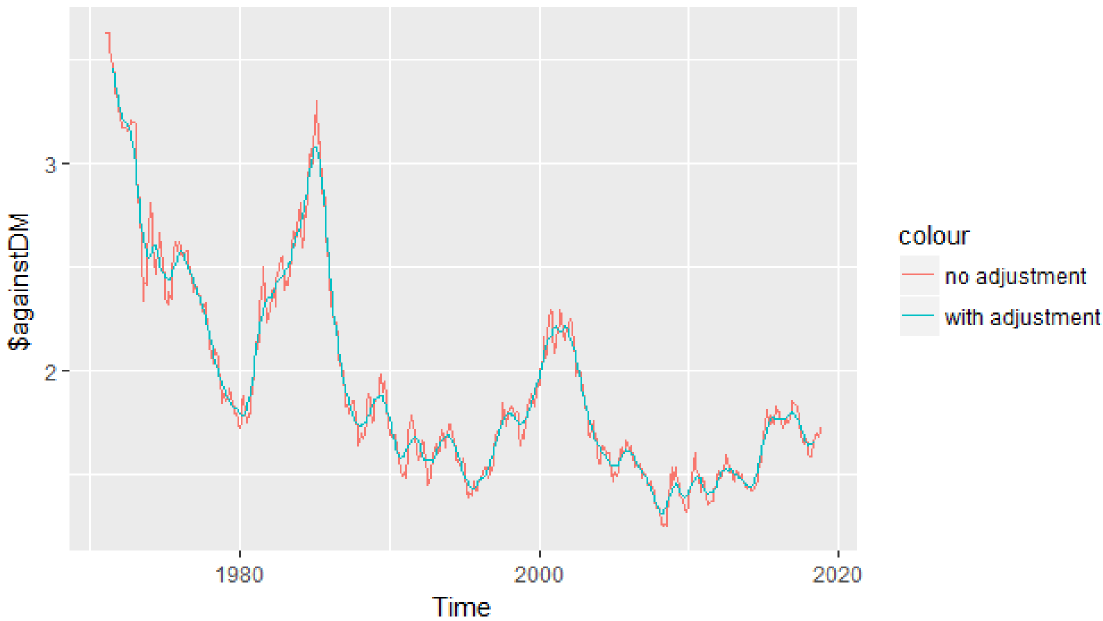

Figure 2: comparison of exchaneg rate

3 Bivariate Specification

The main equation for this specification is:

$$s_t = \beta_0 + \beta_1(p_t - p_t^*) + \epsilon_t \tag{3}$$

From equation(3) we want to check if β_1 is statistically significantly different than

1 and in such a case we say that there is evidence of PPP. Other factors affecting the exchange rate, eg. tariffs, trade restriction etc. are reflected by β_0 which can be different from 0(zero). Before we can run equation(3) it is essential that the variables are stationary and we run the Augmented Dickey Fuller test to check for stationarity and we find that the series are non-stationary. To solve this we take the first difference The data is stationary at 10% significance level. It can be said that the order of integration of the variables is 1, ie. I(1). The result for the stationarity of the series are shown in Figure (3a, 3b, 3c).

```
Augmented Dickey-Fuller test for d_l_CPI_US
including 11 lags of (1-L)d_l_CPI_US
(max was 12, criterion AIC)
sample size 533
unit-root null hypothesis: a = 1

  with constant and trend
  model: (1-L)y = b0 + b1*t + (a-1)*y(-1) + ... + e
  estimated value of (a - 1): -0.279849
  test statistic: tau_ct(1) = -4.3475
  asymptotic p-value 0.002592
  1st-order autocorrelation coeff. for e: 0.000
  lagged differences: F(11, 519) = 4.005 [0.0000]
```

(a) CPI_{US}

```
Augmented Dickey-Fuller test for d_l_CPI_G
including 11 lags of (1-L)d_l_CPI_G
(max was 12, criterion AIC)
sample size 533
unit-root null hypothesis: a = 1

  with constant and trend
  model: (1-L)y = b0 + b1*t + (a-1)*y(-1) + ... + e
  estimated value of (a - 1): -0.0144937
  test statistic: tau_ct(1) = -4.76521
  asymptotic p-value 0.0005001
  1st-order autocorrelation coeff. for e: 0.009
  lagged differences: F(11, 519) = 108.798 [0.0000]
```

(b) CPI_G

```
Augmented Dickey-Fuller test for d_l_USagDM
including 3 lags of (1-L)d_l_USagDM
(max was 12, criterion AIC)
sample size 541
unit-root null hypothesis: a = 1

  with constant and trend
  model: (1-L)y = b0 + b1*t + (a-1)*y(-1) + ... + e
  estimated value of (a - 1): -0.0299226
  test statistic: tau_ct(1) = -3.56143
  asymptotic p-value 0.03317
  1st-order autocorrelation coeff. for e: -0.003
  lagged differences: F(3, 535) = 28.979 [0.0000]
```

(c) Nominal Exchange Rate

Figure 3: Augmented Dickey Fuller Test

Running the regression as in equation(3) gives the following result:

$$\widehat{\text{d\_l\_USagDM}} = \underset{(0.00030755)}{-0.00101652} - \underset{(0.099376)}{0.104190}\,\text{CPI\_diff}$$
$$T = 545 \quad \bar{R}^2 = 0.0002 \quad F(1,543) = 1.0992 \quad \hat{\sigma} = 0.0066407$$
$$\text{(standard errors in parentheses)}$$

Results obtained from doing the ADF test on the residuals of the above equation shows stationarity. The coefficient of CPI_{diff} which is $\hat{\beta}_1$ from equation(3) is not statistically significant as can be seen from the above equation (t stat = 1.04 < 1.96). Thus given the coefficient of CPI_{diff} is not statistically significant and that the residual is stationary we state that there is evidence of PPP as a valid phenomenon. Figure (4) shows the ADF test of the residual of this model.

```
Augmented Dickey-Fuller test for uhat1
including 3 lags of (1-L)uhat1
(max was 18, criterion AIC)
sample size 541
unit-root null hypothesis: a = 1

  test with constant
  model: (1-L)y = b0 + (a-1)*y(-1) + ... + e
  estimated value of (a - 1): -0.0304668
  test statistic: tau_c(1) = -3.56325
  asymptotic p-value 0.00653
  1st-order autocorrelation coeff. for e: -0.002
  lagged differences: F(3, 536) = 24.301 [0.0000]
```

Figure 4: ADF test of resdual from Bivariate specification

4 Univariate Specification

Equation(4) states the model for this specification:

$$\epsilon = p_t^* - p_t + s_t \tag{4}$$

where $\beta_0 = 0$ and $\beta_1 = 1$ is assumed for this model. To check for the evidence for PPP we have to examine whether ϵ, the real exchange rate, is stationary or not. Doing a stationarity check (Figure 5) on this model shows that real exchange is in fact non-stationary (p-value = 0.223). This essentially means that it follows a random walk. Using knowledge from the work of the Balassa-Samuelson model, we can take the real exchange rate from equation(4) as a random walk that can exist due to sectorial productivity differences across countries which leads to changes in the real exchange rate.

5 Cointegration Analysis

The equation to check for the validity for this analysis is :

$$lnS_t = b_0 + b_1 ln\frac{P_t^D}{P_t^F} + j_t \tag{5}$$

Here, P_t^D and P_t^F is CPI of US and Germany respectively as used in this paper. Equation (5) suggests a long run equilibrium relationship exist and this supports the evidence of PPP. Our series is I(1) and I can test for this long run relationship using Johansen Cointegration test. The test is performed using R and I start with finding the VAR(p) model that best suits this relationships as Figure (6) shows.

```
Augmented Dickey-Fuller test for RealExR
including 12 lags of (1-L)RealExR
(max was 18, criterion AIC)
sample size 533
unit-root null hypothesis: a = 1

  with constant and trend
  model: (1-L)y = b0 + b1*t + (a-1)*y(-1) + ... + e
  estimated value of (a - 1): -0.00227553
  test statistic: tau_ct(1) = -2.73308
  asymptotic p-value 0.223
  1st-order autocorrelation coeff. for e: 0.003
  lagged differences: F(12, 518) = 214.694 [0.0000]
```

Figure 5: ADF test of real exchange rate

Using AIC I choose a VAR(4) model and run the johansen cointegration test and results are shown in Figure (7). The components of the largest eigenvector admits the important property of forming the coefficients of a linear combination of time series to produce a stationary portfolio. The trace statistics shows that for $H_o : r = 0$ we reject the null hypothesis of no cointegration as test statistics is greater than the critical value of 30.45 at 1% significance level.Howeve for $H_o : r <= 1$ I see that test statistics is less than critical value of 16.26 at 1% significance level. Thus I fail to reject the null and conclude that there is at most 1 cointegrating equation and there is evidence of PPP having a long run equilibrium relationship. As a result we can run the Vector Error correction model.However it must be noted that at 5% significance level I do do not see any cointegrating relationship and given our series is I(1) at 5% significance level we assert that there is indeed no cointegrating relationship. R codes are given below where lnS_t is logarithmic form of nominal exchange rate(S_t), $lnCPI$ is the logarithmic form of P_t^D/P_t^F:

```
> library('readxl')
> library('ggplot2')
> library("foreign")
> library("forecast")
> library('tseries')
> library('urca')
> library('vars')
> DATA <- read_excel("G:/mahmood/DAL/Time series/Project two/DATA.xlsx")
> #DATA<-DATA[,-7]
> #DATA$lnS_t = log(DATA$USagDM)
> #DATA$lnCPI = log(DATA$CPI_US) - log(DATA$CPI_G)
```

```
> attach(DATA)
> newDATA<-cbind(lnS_t,lnCPI)
> VARselect(newDATA, lag.max = 10, type = "const")

$selection
AIC(n)  HQ(n)  SC(n) FPE(n)
     4      3      3      4

$criteria
                   1             2             3             4             5
AIC(n) -2.190271e+01 -2.532142e+01 -2.541742e+01 -2.541787e+01 -2.541523e+01
HQ(n)  -2.188395e+01 -2.529015e+01 -2.537364e+01 -2.536159e+01 -2.534643e+01
SC(n)  -2.185476e+01 -2.524150e+01 -2.530552e+01 -2.527400e+01 -2.523938e+01
FPE(n)  3.074482e-10  1.007038e-11  9.148659e-12  9.144516e-12  9.168811e-12
                   6             7             8             9            10
AIC(n) -2.540052e+01 -2.539201e+01 -2.538354e+01 -2.537284e+01 -2.539082e+01
HQ(n)  -2.531922e+01 -2.529820e+01 -2.527723e+01 -2.525401e+01 -2.525949e+01
SC(n)  -2.519271e+01 -2.515223e+01 -2.511179e+01 -2.506911e+01 -2.505512e+01
FPE(n)  9.304698e-12  9.384293e-12  9.464246e-12  9.566253e-12  9.395957e-12

> cointest=ca.jo(newDATA, type="trace", K=4, ecdet="trend", spec="longrun")
> summary(cointest)

######################
# Johansen-Procedure #
######################

Test type: trace statistic , with linear trend in cointegration

Eigenvalues (lambda):
[1]  3.395473e-02  2.508996e-02 -6.938894e-18

Values of teststatistic and critical values of test:

          test 10pct  5pct  1pct
r <= 1 | 13.77 10.49 12.25 16.26
r = 0  | 32.50 22.76 25.32 30.45

Eigenvectors, normalised to first column:
(These are the cointegration relations)

             lnS_t.l4      lnCPI.l4    trend.l4
lnS_t.l4 1.0000000000  1.0000000000   1.0000000
lnCPI.l4 1.2867335221 -0.3852757766 -90.0622063
trend.l4 0.0002573131  0.0004212801   0.1184048
```

```
Weights W:
(This is the loading matrix)

            lnS_t.l4      lnCPI.l4      trend.l4
lnS_t.d -0.001018486 -0.0006706044 -8.793195e-18
lnCPI.d -0.002151440  0.0012546934  3.865516e-19
```

6 Conclusion

This paper looked at the different methods of testing for the evidence of PPP. Using Bivariate specification we found evidence of PPP but with the Univariate specification, we saw that the real exchange rate follows a random walk and this goes against the evidence of PPP. On the other hand, using Cointegration analysis we do see the evidence of PPP having a long-run equilibrium relationship between lnS_t and CPI_{diff} only at 1% significance level and not at 5% .